OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4756

Further Methods for Advanced Mathematics (FP2)

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Monday 10 January 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (54 marks)

Answer all the questions

1 **(a)** A curve has polar equation $r = 2(\cos\theta + \sin\theta)$ for $-\frac{1}{4}\pi \leqslant \theta \leqslant \frac{3}{4}\pi$.

(i) Show that a cartesian equation of the curve is $x^2 + y^2 = 2x + 2y$. Hence or otherwise sketch the curve. **[5]**

(ii) Find, by integration, the area of the region bounded by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{2}\pi$. Give your answer in terms of π. **[7]**

(b) **(i)** Given that $\mathrm{f}(x) = \arctan\left(\frac{1}{2}x\right)$, find $\mathrm{f}'(x)$. **[2]**

(ii) Expand $\mathrm{f}'(x)$ in ascending powers of x as far as the term in x^4.

Hence obtain an expression for $\mathrm{f}(x)$ in ascending powers of x as far as the term in x^5. **[5]**

2 **(a)** **(i)** Given that $z = \cos\theta + \mathrm{j}\sin\theta$, express $z^n + z^{-n}$ and $z^n - z^{-n}$ in simplified trigonometrical form. **[2]**

(ii) By considering $\left(z + z^{-1}\right)^6$, show that

$$\cos^6\theta = \frac{1}{32}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10).$$ **[3]**

(iii) Obtain an expression for $\cos^6\theta - \sin^6\theta$ in terms of $\cos 2\theta$ and $\cos 6\theta$. **[5]**

(b) The complex number w is $8\mathrm{e}^{\mathrm{j}\pi/3}$. You are given that z_1 is a square root of w and that z_2 is a cube root of w. The points representing z_1 and z_2 in the Argand diagram both lie in the third quadrant.

(i) Find z_1 and z_2 in the form $r\mathrm{e}^{\mathrm{j}\theta}$. Draw an Argand diagram showing w, z_1 and z_2. **[6]**

(ii) Find the product $z_1 z_2$, and determine the quadrant of the Argand diagram in which it lies. **[3]**

3 **(i)** Show that the characteristic equation of the matrix

$$\mathbf{M} = \begin{pmatrix} 1 & -4 & 5 \\ 2 & 3 & -2 \\ -1 & 4 & 1 \end{pmatrix}$$

is $\lambda^3 - 5\lambda^2 + 28\lambda - 66 = 0$. **[4]**

(ii) Show that $\lambda = 3$ is an eigenvalue of $\mathbf{M}$, and determine whether or not $\mathbf{M}$ has any other real eigenvalues. **[4]**

(iii) Find an eigenvector, $\mathbf{v}$, of unit length corresponding to $\lambda = 3$.

State the magnitude of the vector $\mathbf{M}^n\mathbf{v}$, where n is an integer. **[5]**

(iv) Using the Cayley-Hamilton theorem, obtain an equation for $\mathbf{M}^{-1}$ in terms of $\mathbf{M}^2$, $\mathbf{M}$ and $\mathbf{I}$. **[3]**

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Solve the equation

$$\sinh t + 7\cosh t = 8,$$

expressing your answer in exact logarithmic form. **[6]**

A curve has equation $y = \cosh 2x + 7\sinh 2x$.

(ii) Using part **(i)**, or otherwise, find, in an exact form, the coordinates of the points on the curve at which the gradient is 16.

Show that there is no point on the curve at which the gradient is zero.

Sketch the curve. **[8]**

(iii) Find, in an exact form, the positive value of a for which the area of the region between the curve, the x-axis, the y-axis and the line $x = a$ is $\frac{1}{2}$. **[4]**

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 A curve has parametric equations

$$x = t + a\sin t, \qquad y = 1 - a\cos t,$$

where a is a positive constant.

(i) Draw, on separate diagrams, sketches of the curve for $-2\pi < t < 2\pi$ in the cases $a = 1$, $a = 2$ and $a = 0.5$.

By investigating other cases, state the value(s) of a for which the curve has

(*A*) loops,

(*B*) cusps. **[7]**

(ii) Suppose that the point P (x, y) lies on the curve. Show that the point P′ $(-x, y)$ also lies on the curve. What does this indicate about the symmetry of the curve? **[3]**

(iii) Find an expression in terms of a and t for the gradient of the curve. Hence find, in terms of a, the coordinates of the turning points on the curve for $-2\pi < t < 2\pi$ and $a \neq 1$. **[5]**

(iv) In the case $a = \frac{1}{2}\pi$, show that $t = \frac{1}{2}\pi$ and $t = \frac{3}{2}\pi$ give the same point. Find the angle at which the curve crosses itself at this point. **[3]**

GCE

Mathematics (MEI)

Advanced GCE

Unit **4756:** Further Methods for Advanced Mathematics

Mark Scheme for January 2011

Any enquiries about publications should be addressed to:

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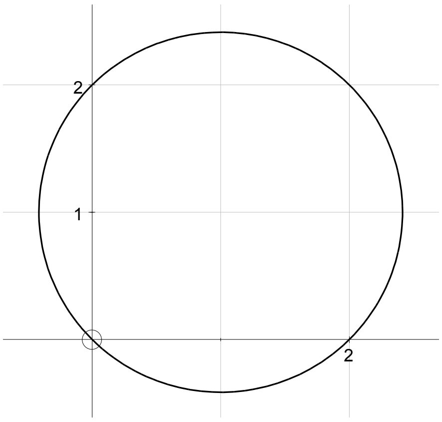
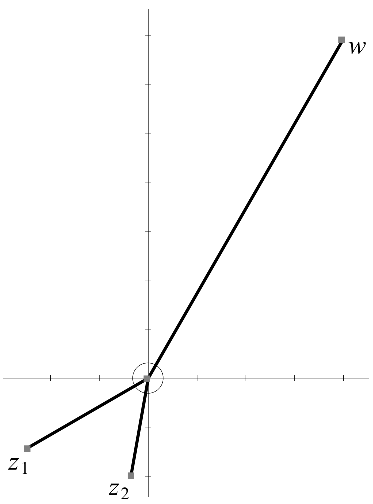
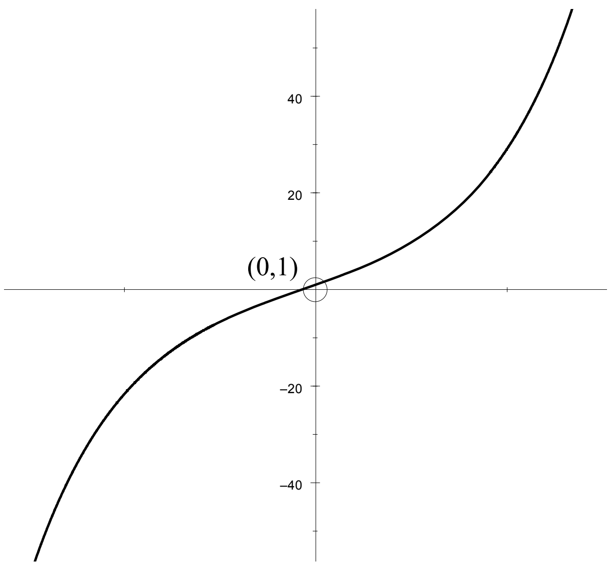
1 (a)(i)	$x = r\cos\theta,\ y = r\sin\theta,\ x^2 + y^2 = r^2$ $r = 2(\cos\theta + \sin\theta)$ $\Rightarrow\ r^2 = 2r(\cos\theta + \sin\theta)$ $\Rightarrow\ x^2 + y^2 = 2x + 2y$ $\Rightarrow\ x^2 - 2x + y^2 - 2y = 0$ $\Rightarrow\ (x-1)^2 + (y-1)^2 = 2$ which is a circle centre (1, 1) radius $\sqrt{2}$ 	M1 A1 (ag) M1 G1 G1 **5**	Using at least one of these Working must be convincing Recognise as circle or appropriate algebra leading to $(x-a)^2 + (y-b)^2 = r^2$ Attempt at complete circle with centre in first quadrant A circle with centre and radius indicated, or centre (1, 1) indicated and passing through (0, 0), or (2, 0) and (0, 2) indicated and passing through (0, 0)
(ii)	Area $= \frac{1}{2}\int_0^{\frac{\pi}{2}} r^2 d\theta$ $= 2\int_0^{\frac{\pi}{2}} (\cos\theta + \sin\theta)^2\, d\theta$ $= 2\int_0^{\frac{\pi}{2}} (\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta)\, d\theta$ $= 2\int_0^{\frac{\pi}{2}} (1 + 2\sin\theta\cos\theta)\, d\theta$ $= 2\left[\theta - \frac{1}{2}\cos 2\theta\right]_0^{\frac{\pi}{2}}$ or $2\left[\theta + \sin^2\theta\right]_0^{\frac{\pi}{2}}$ etc. $= 2\left(\left(\frac{\pi}{2} + \frac{1}{2}\right) - \left(0 - \frac{1}{2}\right)\right)$ $= \pi + 2$	 M1 M1 A1 A2 M1 A1 **7**	 Integral expression involving r^2 in terms of θ Multiplying out $\cos^2\theta + \sin^2\theta = 1$ used Correct result of integration with correct limits. Give A1 for one error Substituting limits. Dep. on both M1s Mark final answer
(b)(i)	$f'(x) = \frac{1}{2}\frac{1}{\left(1 + \frac{1}{4}x^2\right)} = \frac{2}{4 + x^2}$	M1 A1 **2**	Using Chain Rule Correct derivative in any form
(ii)	$f'(x) = \frac{1}{2}\left(1 + \frac{1}{4}x^2\right)^{-1} = \frac{1}{2}\left(1 - \frac{1}{4}x^2 + \frac{1}{16}x^4 - ...\right)$ $= \frac{1}{2} - \frac{1}{8}x^2 + \frac{1}{32}x^4 - ...$ $\Rightarrow f(x) = \frac{1}{2}x - \frac{1}{24}x^3 + \frac{1}{160}x^5 - ... + c$ But $c = 0$ because arctan(0) = 0	M1 A1 M1 A1 A1 **5**	Correctly using binomial expansion Correct expansion Integrating at least two terms Independent **19**

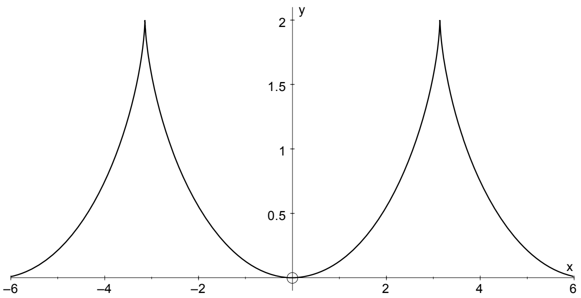
2 (a)(i)	$z^n + z^{-n} = 2\cos n\theta$ $z^n - z^{-n} = 2\text{j}\sin n\theta$	B1 B1 **2**	
(ii)	$(z + z^{-1})^6 = z^6 + 6z^4 + 15z^2 + 20 + 15z^{-2} + 6z^{-4} + z^{-6}$ $= z^6 + z^{-6} + 6(z^4 + z^{-4}) + 15(z^2 + z^{-2}) + 20$ $\Rightarrow 64\cos^6\theta = 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$ $\Rightarrow \cos^6\theta = \frac{1}{32}\cos 6\theta + \frac{3}{16}\cos 4\theta + \frac{15}{32}\cos 2\theta + \frac{5}{16}$ $\Rightarrow \cos^6\theta = \frac{1}{32}\left(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10\right)$	M1 M1 A1 (ag) **3**	Expanding $(z + z^{-1})^6$ Using $z^n + z^{-n} = 2\cos n\theta$ with $n = 2, 4$ or 6. Allow M1 if 2 omitted, etc.
(iii)	$(z - z^{-1})^6 = z^6 + z^{-6} - 6(z^4 + z^{-4}) + 15(z^2 + z^{-2}) - 20$ $\Rightarrow -64\sin^6\theta = 2\cos 6\theta - 12\cos 4\theta + 30\cos 2\theta - 20$ $\Rightarrow -\sin^6\theta = \frac{1}{32}\cos 6\theta - \frac{3}{16}\cos 4\theta + \frac{15}{32}\cos 2\theta - \frac{5}{16}$ $\Rightarrow \cos^6\theta - \sin^6\theta = \frac{1}{16}\cos 6\theta + \frac{15}{16}\cos 2\theta$	B1 M1 A1 M1 A1	Using (i) as in part (ii) Correct expression in any form Attempting to add or subtract
	OR $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$ B1 $16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$ M1 $\Rightarrow \cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$ A1 $\cos^6\theta - \sin^6\theta = 2\cos^6\theta - 3\cos^4\theta + 3\cos^2\theta - 1$ $\Rightarrow = \frac{1}{16}\cos 6\theta + \frac{15}{16}\cos 2\theta$ M1A1	**5**	This used Obtaining an expression for $\cos^4\theta$ Correct expression in any form Attempting to add or subtract
(b)(i)	${z_1}^2 = 8e^{\frac{j\pi}{3}} \Rightarrow z_1 = 2\sqrt{2}e^{j\left(\frac{\pi}{6}+\pi\right)}$ $= 2\sqrt{2}e^{\frac{j7\pi}{6}}$ ${z_2}^3 = 8e^{\frac{j\pi}{3}} \Rightarrow z_2 = 2e^{j\left(\frac{\pi}{9}+\frac{4\pi}{3}\right)}$ $= 2e^{\frac{j13\pi}{9}}$ 	M1 A1 M1 A1 G1 G1 **6**	Correctly manipulating modulus and argument $\sqrt{8}$, $\frac{7\pi}{6}$ or $-\frac{5\pi}{6}$. Condone $r(c + js)$ Correctly manipulating modulus and argument 2, $\frac{13\pi}{9}$ or $-\frac{5\pi}{9}$. Condone $r(c + js)$ Moduli approximately correct Arguments approximately correct Give G1G0 for two points approximately correct
(ii)	$z_1 z_2 = 2\sqrt{2}e^{\frac{j7\pi}{6}} \times 2e^{\frac{j13\pi}{9}}$ $= 4\sqrt{2}e^{j\left(\frac{7\pi}{6}+\frac{13\pi}{9}\right)}$ $= 4\sqrt{2}e^{\frac{j11\pi}{18}}$ Lies in second quadrant	M1 A1 A1 **3**	Correctly manipulating modulus and argument Accept any equivalent form **19**

3 (i)	$\det(\mathbf{M}-\lambda\mathbf{I}) = (1-\lambda)[(3-\lambda)(1-\lambda)+8]$ $+ 4[2(1-\lambda)-2] + 5[8+(3-\lambda)]$ $= (1-\lambda)(\lambda^2-4\lambda+11) + 4(-2\lambda) + 5(11-\lambda)$ $= -\lambda^3+5\lambda^2-15\lambda+11-8\lambda+55-5\lambda = 0$ $\Rightarrow \lambda^3-5\lambda^2+28\lambda-66=0$	M1 A1 M1 A1 (ag) **4**	Obtaining $\det(\mathbf{M}-\lambda\mathbf{I})$ Any correct form Simplification www, but condone omission of = 0
(ii)	$\lambda^3-5\lambda^2+28\lambda-66=0$ $\Rightarrow (\lambda-3)(\lambda^2-2\lambda+22)=0$ $\lambda^2-2\lambda+22=0 \Rightarrow b^2-4ac=-84$ so no other real eigenvalues	M1 A1 M1 A1 **4**	Factorising and obtaining a quadratic. If M0, give B1 for substituting $\lambda = 3$ Correct quadratic Considering discriminant o.e. Conclusion from correct evidence www
(iii)	$\lambda = 3 \Rightarrow \begin{pmatrix}-2 & -4 & 5\\ 2 & 0 & -2\\ -1 & 4 & -2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$ $\Rightarrow -2x-4y+5z=0$ $2x-2z=0$ $-x+4y-2z=0$ $\Rightarrow x=z=k,\ y=\frac{3}{4}k$ $\Rightarrow$ eigenvector is $\begin{pmatrix}4\\3\\4\end{pmatrix}$ $\Rightarrow$ eigenvector with unit length is $\mathbf{v}=\frac{1}{\sqrt{41}}\begin{pmatrix}4\\3\\4\end{pmatrix}$ Magnitude of $\mathbf{M}^n\mathbf{v}$ is 3^n	 M1 M1 A1 B1 B1 **5**	 Two independent equations Obtaining a non-zero eigenvector Must be a magnitude
(iv)	$\lambda^3-5\lambda^2+28\lambda-66=0$ $\Rightarrow \mathbf{M}^3-5\mathbf{M}^2+28\mathbf{M}-66\mathbf{I}=\mathbf{0}$ $\Rightarrow \mathbf{M}^2-5\mathbf{M}+28\mathbf{I}-66\mathbf{M}^{-1}=\mathbf{0}$ $\Rightarrow \mathbf{M}^{-1}=\frac{1}{66}(\mathbf{M}^2-5\mathbf{M}+28\mathbf{I})$	 M1 M1 A1 **3**	 Use of Cayley-Hamilton Theorem Multiplying by $\mathbf{M}^{-1}$ and rearranging Must contain **I** **16**

4 (i)	$\sinh t + 7\cosh t = 8$		
	$\Rightarrow \frac{1}{2}(e^{t} - e^{-t}) + 7 \times \frac{1}{2}(e^{t} + e^{-t}) = 8$	M1	Substituting correct exponential forms
	$\Rightarrow 4e^{t} + 3e^{-t} = 8$		
	$\Rightarrow 4e^{2t} - 8e^{t} + 3 = 0$	M1	Obtaining quadratic in e^{t}
	$\Rightarrow (2e^{t} - 1)(2e^{t} - 3) = 0$	M1	Solving to obtain at least one value of e^{t}
	$\Rightarrow e^{t} = \frac{1}{2}$ or $\frac{3}{2}$	A1A1	Condone extra values
	$\Rightarrow t = \ln(\frac{1}{2})$ or $\ln(\frac{3}{2})$	A1	These two values o.e. only. Exact form
		6	
(ii)	$\frac{dy}{dx} = 2\sinh 2x + 14\cosh 2x$ or $8e^{2x} + 6e^{-2x}$	B1	
	$2\sinh 2x + 14\cosh 2x = 16 \Rightarrow \sinh 2x + 7\cosh 2x = 8$		
	$\Rightarrow 2x = \ln(\frac{1}{2})$ or $\ln(\frac{3}{2}) \Rightarrow x = \frac{1}{2}\ln(\frac{1}{2})$ or $\frac{1}{2}\ln(\frac{3}{2})$	M1 A1	Complete method to obtain an x value Both x co-ordinates in any exact form
	$x = \frac{1}{2}\ln(\frac{1}{2}) \Rightarrow y = -4$ $\quad(\frac{1}{2}\ln(\frac{1}{2}), -4)$		
	$x = \frac{1}{2}\ln(\frac{3}{2}) \Rightarrow y = 4$ $\quad(\frac{1}{2}\ln(\frac{3}{2}), 4)$	B1	Both y co-ordinates
	$\frac{dy}{dx} = 0 \Rightarrow 2\sinh 2x + 14\cosh 2x = 0$		
	$\Rightarrow \tanh 2x = -7$ or $e^{4x} = -\frac{3}{4}$ etc.	M1	Any complete method
	No solutions because $-1 < \tanh 2x < 1$ or $e^{x} > 0$ etc.	A1 (ag)	www
		G1 G1	Curve (not st. line) with correct general shape (positive gradient throughout) Curve through (0, 1). Dependent on last G1
		8	
(iii)	$\int_0^a (\cosh 2x + 7\sinh 2x)\,dx = \frac{1}{2}$	M1	Attempting integration
	$\Rightarrow \left[\frac{1}{2}\sinh 2x + \frac{7}{2}\cosh 2x\right]_0^a = \frac{1}{2}$	A1	Correct result of integration
	$\Rightarrow \left(\frac{1}{2}\sinh 2a + \frac{7}{2}\cosh 2a\right) - \frac{7}{2} = \frac{1}{2}$		
	$\Rightarrow \sinh 2a + 7\cosh 2a = 8$		
	$\Rightarrow 2a = \ln(\frac{1}{2})$ or $\ln(\frac{3}{2}) \Rightarrow a = \frac{1}{2}\ln(\frac{1}{2})$ or $\frac{1}{2}\ln(\frac{3}{2})$	M1	Using both limits and a complete method to obtain a value of a
	$\Rightarrow a = \frac{1}{2}\ln(\frac{3}{2})$ $\quad(\frac{1}{2}\ln(\frac{1}{2}) < 0)$	A1	Must reject $\frac{1}{2}\ln(\frac{1}{2})$, but reason need not be given
		4	**18**

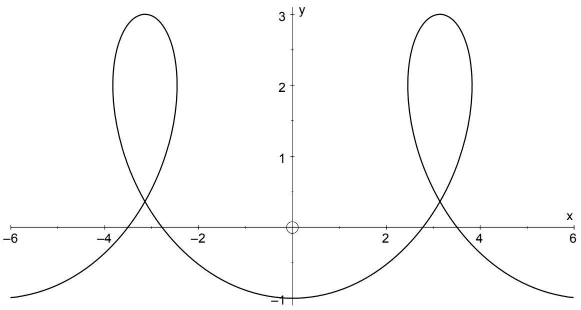
<table>
<tr><td>5 (i)</td><td>

$a = 1$



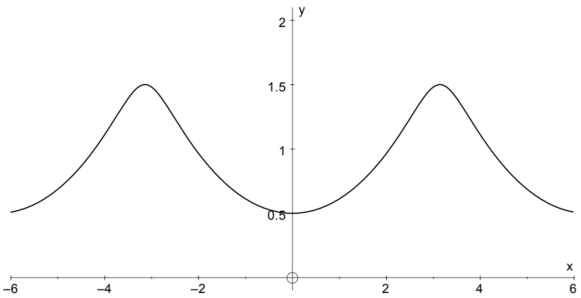
</td><td>G1</td><td></td></tr>
<tr><td></td><td>

$a = 2$



</td><td>G1</td><td></td></tr>
<tr><td></td><td>

$a = 0.5$



</td><td>G1</td><td></td></tr>
<tr><td></td><td></td><td>M2</td><td>Evidence s.o.i. of further investigation</td></tr>
<tr><td>(A)</td><td>Loops when $a > 1$</td><td>A1</td><td></td></tr>
<tr><td>(B)</td><td>Cusps when $a = 1$</td><td>A1
7</td><td></td></tr>
<tr><td>(ii)</td><td>If $x \to -x$, $t \to -t$
but $y(-t) = y(t)$
Curve is symmetrical in the y-axis</td><td>M1
A1 (ag)
B1
3</td><td>Considering effect on t
Effect on y</td></tr>
<tr><td>(iii)</td><td>$\frac{dy}{dx} = \frac{a\sin t}{1 + a\cos t}$
$\frac{dy}{dx} = 0 \Rightarrow a \sin t = 0 \Rightarrow t = 0$ and $\pm\pi$
$t = 0 \Rightarrow$ T.P. is $(0, 1 - a)$
$t = \pm\pi \Rightarrow$ T.P. are $(\pm\pi, 1 + a)$</td><td>M1
A1
A1
A1
A1
5</td><td>Using Chain Rule

Values of t

Both, in any form</td></tr>
<tr><td>(iv)</td><td>$a = \frac{\pi}{2}$: both $t = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ give the point $(\pi, 1)$
Gradients are a and $-a$ (or $\frac{\pi}{2}$ and $-\frac{\pi}{2}$)
Hence angle is $2 \arctan(\frac{\pi}{2}) \approx 2.01$ radians</td><td>B1 (ag)

M1
A1
3</td><td>Verification

Complete method for angle
Accept 115° (or 65°)
18</td></tr>
</table>

4756 Further Methods for Advanced Mathematics

General Comments

After an increase last year, the entry for this paper returned to about the level seen in January 2009. The mean mark was slightly up compared to January 2010, and once again there was a great deal of very good work, with nearly a quarter of candidates scoring 60 marks or more, and only 5% of candidates scoring fewer than 20 marks.

Having said this there were, once again, a number of rather worrying errors, which sometimes appeared even in otherwise good scripts. These included: incorrect assertions and deductions (in Q3, "$\sqrt{-84}$ is negative", or "the quadratic does not factorise, hence it has no real roots"; in Q4 "the curve has no turning points, so it is a straight line"); horrendous algebra (in Q1 alone $(\cos\theta + \sin\theta)^2 = \cos^2\theta + \sin^2\theta$, $\sqrt{x^2+y^2} = x + y$ and $\frac{1}{2+\frac{1}{2}x^2} = \frac{1}{2} + 2x^{-2}$ all appeared) and "original" laws of logarithms (in Q4, $\ln(4e^t + 3e^{-t}) = \ln 8 \Rightarrow t = \ln 8$). In general, candidates were extremely competent when dealing with standard processes, such as finding roots of complex numbers in Q2, finding the characteristic equation of a matrix, finding eigenvectors and using the Cayley-Hamilton Theorem in Q3, and solving the hyperbolic equation in Q4, but the standard of work declined significantly when they were presented with unfamiliar situations, or expected to transfer knowledge from other units. This was particularly evident in Q1(b) which required the binomial theorem from C4, and especially in Q3(iii), where very few candidates were able to exhibit an eigenvector of unit length.

Question 3 was the best done question, with Questions 1 and 4 scoring the lowest. Question 5 (Investigations of Curves) was attempted by only a handful of candidates. There was a little evidence of time trouble, usually affecting candidates who had used very inefficient methods to answer some parts of questions: this was particularly evident in Q1(b), Q2(a)(ii) and (iii) and in some parts of Q4.

Presentation varied from the admirable to the execrable; this time there were a few scripts which were extremely hard to read. Some candidates fitted all their answers into the standard eight-page answer book; others used up to three such books, and there were a few who insisted on presenting graphs and Argand diagrams on separate pieces of graph paper.

Comments on Individual Questions

1) **(a)** Many candidates converted the curve from polars to Cartesians concisely and elegantly. Others used the given equations to obtain $x = 2(\cos\theta + \sin\theta)\cos\theta$, $y = 2(\cos\theta + \sin\theta)\sin\theta$ and tried to show that $x^2 + y^2 = 2x + 2y$ from these, sometimes successfully. A few produced "equations" such as $r = x^2 + y^2$, $x = \cos\theta$, $y = \sin\theta$.

The curve was usually recognised as a circle and frequently placed correctly, with all, or most, of the desired information indicated. Only a few candidates rearranged the Cartesian equation to find the centre and radius of the circle, with the majority producing a table of values of θ and r. Some of the sketches were very, very small.

The method for finding the area of a polar curve by integration was well known, with only a very few candidates attempting to integrate r rather than r^2. Then, having expanded $(\cos\theta + \sin\theta)^2$, most candidates obtained $\cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta$. The identity $\cos^2\theta + \sin^2\theta = 1$ was then commonly ignored, with both

$\cos^2\theta$ and $\sin^2\theta$ being expressed in terms of cos 2θ. The integration often resulted in a sign error, and sometimes both limits were not used. These faults led to the correct answer being seen relatively rarely.

(b) The derivative of arctan($\frac{1}{2}x$) was frequently given as $\dfrac{1}{1+\frac{1}{4}x^2}$.

The intention in the second part here was that candidates should expand $\dfrac{2}{4+x^2}$ by the binomial theorem, and integrate the result to obtain a series for f(x), considering the arbitrary constant. Only the very best candidates got to the very end, with very few showing that $c = 0$. Knowledge of the binomial theorem, and especially how to deal with the 4 in $\left(4+x^2\right)^{-1}$, was by no means universal. A fairly large number tried to deal with the question by repeated differentiation. If this included valid methods, it was credited, but it rarely did.

2) (a)Many candidates spent a great deal of time deriving, rather than writing down, the expressions of $z^n + z^{-n}$ and $z^n - z^{-n}$ in simplified trigonometric form, and their answers were not always correct.

Obtaining the given expression for $\cos^6\theta$ was often done well, but there were many failed attempts. Often the 2s disappeared from, for example, $z^6 + z^{-6} = 2\cos 6\theta$. Candidates who did this often found it convenient to assume that $2^6 = 32$. Others ignored the advice in the question and considered $(\cos\theta + j\sin\theta)^6$.

Obtaining an expression for $\cos^6\theta - \sin^6\theta$ caused a great deal of trouble to some candidates, while others used elegant and efficient methods to produce a correct answer in a few lines. The usual approach was to try to obtain an expression as in (ii) for $\sin^6\theta$. Candidates started with $(z - z^{-1})^6$ and expanded it, with varying degrees of success, and often obtained sines of multiple angles rather than cosines when using (i). Others lost a minus sign, or gained a j, when expanding $(2j\sin\theta)^6$. There were many alternative methods, but those who used trigonometric identities alone were rarely successful, and often wasted several pages over the attempt.

(b) Finding the square and cube roots of $8e^{j\pi/3}$ was done very efficiently by the majority of candidates, although a little more trouble was experienced in locating the roots in the desired quadrant, and a few weaker candidates failed to divide π/3 by 2 or by 3. The Argand diagram was usually adequate, although some forgot to plot w, and when it did appear, it was sometimes closer to the real than to the imaginary axis.

The method for finding the product z_1z_2 from the exponential forms was well known, and many candidates were able to correctly deduce the quadrant in which the product lay.

3) **(i)** Virtually every candidate knew how to obtain the characteristic equation, although there were a few sign errors. Most expanded by the first row or the first column, and Sarrus' method was also popular. One candidate used elementary row operations to produce a very elegant solution. "Invisible brackets" around the $1 - \lambda$ were condoned.

(ii) This was also well done. The quadratic factor was usually obtained correctly, and the absence of real roots deduced, although there were a few careless errors at this stage.

(iii) The method for producing an eigenvector was well known, although some candidates used $(\mathbf{M} - \lambda\mathbf{I})\mathbf{v} = \mathbf{v}$ or $\lambda\mathbf{v}$. Those who obtained $3x = 4y$ often went on to give $x = 3$, $y = 4$ in the eigenvector. But there were very many correct directions.

The rest of this part of the question was unpopular with candidates. Only a very few were able to produce a unit vector in the direction of their eigenvector. Most ignored the instruction completely. Slightly more success was obtained with the magnitude of $\mathbf{M}^n\mathbf{v}$, although some candidates, seeing $\mathbf{M}^n$, were determined to use the result $\mathbf{M}^n = \mathbf{P}\mathbf{D}^n\mathbf{P}^{-1}$, wasting a lot of time as a result.

(iv) The Cayley-Hamilton Theorem and its application here were well known, although there were many sign errors, and the identity matrix **I** was sometimes omitted from the answer.

4) (i) There were many fully correct solutions to this part. No candidate who used the exponential definitions of sinh and cosh mixed them up, which was very pleasing. The vast majority of candidates used this method, but four other successful methods were seen, including a very impressive solution using $r\sinh(x + \alpha)$, which assisted the candidate in part (ii). Weaker candidates could not obtain, or cope with, a quadratic in e^t, often inventing spurious laws of logarithms.

(ii) Most candidates could differentiate $\cosh 2x + 7\sinh 2x$, although a few multiplied by ½ rather than 2. Then the intention was that, having set this expression equal to 8, candidates would use the answer to part (i). Many did, although they sometimes failed to use the 2 in $2x$, but others started again, spending a great deal of time doing so and rarely obtaining the correct answer. The y co-ordinates were often forgotten. Then candidates were asked to show that there was no point on the curve with gradient 0. Many resorted to exponentials and observed that $4e^{2x} + 3e^{-2x}$ would never be zero, although often this was just asserted without supporting evidence, and many incorrect statements were seen. Having proved (or not) that there were no turning points, many candidates deduced that the "curve" must be a straight line, and some said as much. However, many did produce the correct shape, although it often passed through the origin rather than (0, 1).

(iii) The majority tried to integrate, often successfully, although a few multiplied by 2 rather than ½. Then once again part (i) could be invoked, and often was, although the negative solution was often given as well. Many started again; if errors were made, this often led to nasty quadratics with hideous surds as their roots.

5) There were a handful of attempts at this question, of which only one made significant progress past part (i).

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